ECONOMIC AND REGIONAL STUDIES

STUDIA EKONOMICZNE I REGIONALNE

AXIOMATIC FORMULATION OF THE OPTIMAL TRANSACTION COST THEORY IN THE LEGAL PROCESS THROUGH COBB-DOUGLAS OPTIMIZATION

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Abstract

Subject and purpose of work: This paper uses Cobb-Douglas optimization to formulate an optimal transaction cost algorithm within the constraint of a generalized legal framework.
Materials and methods: The author has adopted a Lagrangian approach to formulate the social utility function, then, from a set of legally allowed strategies established the Karush-Kuhn-Tucker conditions for the legal game so as to find the optimal parameters within the social utility function. Finally, the optimal transaction cost algorithm was developed.
Results: The Bordered Hessian Matrix from the partial differentials of the social utility function showed that there is a particular parameter within the social utility function which describes the optimal transaction cost. An adjustment of this parameter is essential in mechanism design for legal games.
Conclusions: The author has shown how transaction costs influence the set of strategies played by players in a legal game, and has described the essence of a social utility function and how it can be optimized.

Keywords: transaction costs, legal process, Cobb-Douglas optimization, game theory, Lagrangian approach

Introduction

In a legal dispute, parties engage in a series of negotiations so as to arrive at a reasonable settlement. The parties need to present a fair and reasonable bargain in order to induce the WTA (willingness to accept) of the plaintiff and the WTP (willingness to pay) of the defendant. Cooperation can only be attained when the WTA of the plaintiff is less than or equal to the WTP of the defendant, thus WTA≤WTP. From an economic perspective, the legal process can be considered as a market place of buying and selling claims. High transaction costs decrease the reasonable bargain thereby making cooperation more appealing to the defendant. On the other hand, low or zero transaction costs means the reasonable bargain is only dependent on the expected gain from winning at trial and the settlement benefit thereby

making cooperation more appealing to the plaintiff. Hence, we need to find a way of adjusting the number of settlements with the number of trials in order to maximize the social utility value. This paper uses Cobb-Douglas optimization to formulate an optimal transaction cost algorithm considering the confinement of a generalized legal framework.

In the paper titled "*Mechanism of Instrumental Game Theory in the Legal Process via Stochastic Options Pricing Induction*" (Osei Bonsu, Chen, 2020), it was deduced that the reasonable bargain in a legal dispute can be represented as a function of the expectation benefit and the transaction costs as follows:
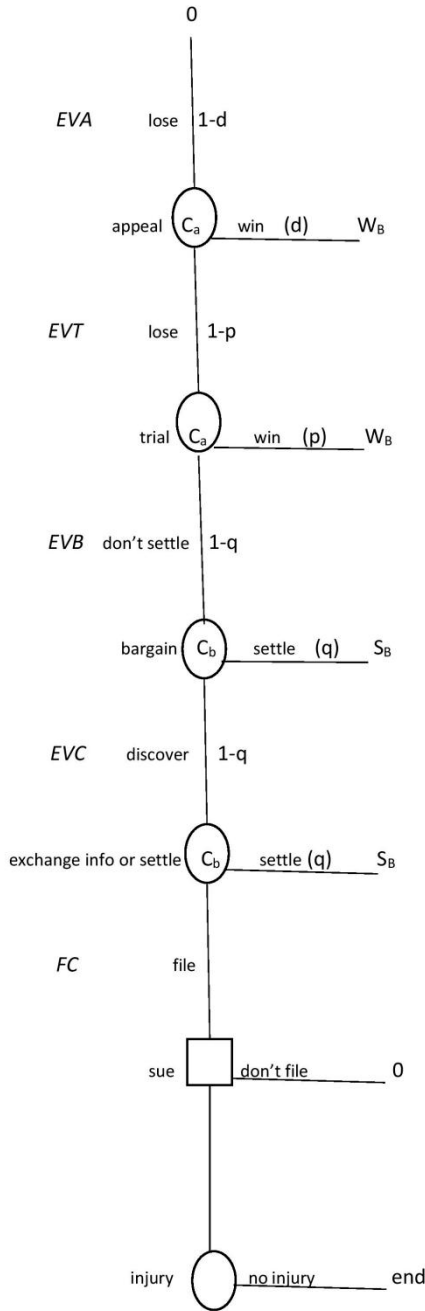
$$R_B = \frac{1}{2}\left[pW_B + S_B\right] - \frac{1}{2}\left[C_a + 3C_b\right] \tag{1}$$

deduced from figure 1 below,

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Figure 1. Stages in the legal process
Source: Osei Bonsu and Chen 2020.

R_B is the reasonable bargain, p is the probability of the plaintiff winning the trial, W_B is the winning benefit from the trial for the plaintiff, C_a is the administration costs, C_b is the bargaining costs and S_B is the settlement benefit for the plaintiff as a result of bargaining.

The transaction costs comprise of two main parts; C_b, bargaining costs (including negotiations, lawyer fees, information exchange etc.) and C_a, administration costs (trial costs, including court fees, lawyer fees, time etc.).

Assuming the parties involved are rational, there are four possible scenarios which can occur if the thresholds of the transaction costs are labeled as high or low;

1. High C_b + High C_a

In this case the overall transaction costs are too high so the plaintiff will choose trial over settlement since the expectation at trial is greater than the bargaining benefit.

2. High C_b + Low C_a

In this case the plaintiff will definitely choose trial since it is obvious that the overall benefit from trial is greater than settlement.

3. Low C_b + High C_a

In this case the plaintiff will choose settlement if the expectation at trial minus C_a is less than settlement minus C_b.

4. Low C_b + Low C_a

In this case the plaintiff will still choose trial because the expectation at trial is greater than settlement.

So it is evident that it is only in the third case will the plaintiff consider settlement. All the other cases will lead to trial thereby increasing the number of trials, which is not the optimal outcome for society.

In the third scenario, however, even though more cases result in settlements, another problem arises.

To explain the problem, first let us look at the Learned Hands Rule (Kim, 2013); this is an algebraic formula, simply written as B = PL , according to which liability depends on the relation between investment in precaution B, the product of the probability P, and magnitude of harm L resulting from the harm caused by the defendant. If PL exceeds B, then the defendant should be liable.

If more cases end up in settlements, more injurers will exercise less precaution since they know there is not much to lose if the case is filed. This will result in more injuries, and subsequently more cases. More cases means more trials even though a lower percentage of cases end up at trial, the aggregate number of trials increases, which is also not a socially optimal outcome.

Hence we can infer that transaction cost theory in the legal process is not as straightforward as previously discussed in law and economics literature. This is why a more rigorous approach is needed to find a way to estimate not necessarily high or low but an optimal transaction cost algorithm whose parameters and superficial assumptions can be adjusted based on individual cases.

In this paper we will show the effect of administration costs on the outcomes of a legal dispute and propose a methodology for estimating the optimal transaction cost in the legal process in order to obtain a social optimum.

The set of strategies in a cooperative legal game

The players in a cooperative legal game jointly choose a set of moral codes m from a set of all possible codes M (Flores Borda, 2011),

i.e $m \in M$.

The set m contains a set of legal rules R such that R is an element of M. Then the players choose a specific legal rule r from R, i.e $r \in R$.

Assuming that the legal rules are supposed to maximize their social utility function W, then for the sake of cooperation each player i has to choose strategy s_i from a set of strategies S,

i.e $s_i \in S$.

In this case we can find W_{max} which is the social maximum. We should note that there is a subset of allowed strategies $P(r)$ under legal rule r within S. So each player i must choose a strategy within $P(r)$,

i.e $s_i \in P(r)$.

However, we know that in the real world players do not always follow the legal rules. This is why we need to set transaction costs in such a way that the utility U where s_i is an element of $P(r)$ will be greater than W where s_i is an element of S.

Utility maximization under constrained strategies

From equation 1 let

$$Pc = \frac{1}{2}\left[pW_B + S_B\right]$$

and

$$Lc = \frac{1}{2}\left[C_a + 3C_b\right]$$

where Pc is the expectation benefit component and Lc is the transaction cost component. We can say that $L_C + R_B = Pc$. Then the utility function U can be defined as a Cobb-Douglas utility (Sandler, Ivanov, Lifshits, Diligenskaya, 2015) (Panhans, 2021),

$$U(L_C, R_B) = L_C^\alpha R_B^\beta \qquad (2)$$

where alpha and beta are some real numbers.

Since we are working within the legal confinement, our feasibility constraint is dependent upon the judgment at trial or the settlement benefit. Let Lc and R_B be influenced by the factors p_1 and p_2 respectively, the constraint mechanism results in equation 3 below.

$$p_1 L_C + p_2 R_B \leq P_C \qquad (3)$$

The R_B line has a slope of $-p_1/p_2$, x-intercept of Pc/p_1 and y-intercept of Pc/p_2.

By Lagrangian approach, we can transform the constrained optimization problem into an unconstrained form. Thus,

$$L = U(R_B, L_C) + \lambda[P_C - p_1 LC - p_2 R_B] \qquad (4)$$

where L is the Lagrange function, λ is the Lagrange multiplier.

The resulting first order conditions are;

$$L_{Lc} = \frac{\partial U}{\partial Lc} - \lambda p_1 = 0$$

$$L_{R_B} = \frac{\partial U}{\partial R_B} - \lambda p_2 = 0$$

$$L_\lambda = Pc - p_1 Lc - p_2 R_B = 0$$

$$\Rightarrow$$

$$\frac{\partial U}{\partial Lc} = \lambda p_1$$

$$\frac{\partial U}{\partial R_B} = \lambda p_2 \qquad (5)$$

$$Pc = p_1 Lc + p_2 R_B$$

In terms of the Cobb-Douglas Utility let's start from equation 2, then we can deduce that the marginal rate of substitution is as shown below;

$$MRS = \frac{\alpha R_B}{\beta L_C}$$

$$\Rightarrow \frac{\alpha R_B}{\beta L_C} = \frac{p_1}{p_2} \Rightarrow R_B = \frac{p_1}{p_2} \cdot \frac{\beta}{\alpha} L_C$$

Substituting R_B into the feasibility constraint yields,

$$P_C = p_1 L_C + p_2 (\frac{p_1}{p_2} \frac{\beta}{\alpha} L_C)$$

$$P_C = p_1 L_C + \frac{\beta}{\alpha} p_1 L_C = (\frac{\alpha + \beta}{\alpha}) p_1 L_C \qquad (6)$$

$$L_C = (\frac{\alpha}{\alpha + \beta}) \frac{Pc}{p_1} \Rightarrow p_1 = (\frac{\alpha}{\alpha + \beta}) \frac{Pc}{L_C}$$

Likewise we can say that,

$$R_B = (\frac{\beta}{\alpha + \beta}) \frac{Pc}{p} \Rightarrow p = (\frac{\beta}{\alpha + \beta}) \frac{Pc}{R_B}$$

Substituting p_1 and p_2 into the resulting first order conditions we get,

$$\frac{\partial U}{\partial Lc} = \lambda (\frac{\alpha}{\alpha + \beta}) \frac{Pc}{Lc}$$

$$\frac{\partial U}{\partial R_B} = \lambda (\frac{\beta}{\alpha + \beta}) \frac{Pc}{R_B}$$

$\because U(L_C, R_B) = L_C^\alpha R_B^\beta$ first order partial derivatives result in;

$$\frac{\partial U}{\partial Lc} = \lambda(\frac{\alpha}{\alpha+\beta})\frac{Pc}{Lc} = \alpha L_c^{\alpha-1}R_B^{\beta}$$

$$\frac{\partial U}{\partial R_B} = \lambda(\frac{\beta}{\alpha+\beta})\frac{Pc}{R_B} = \beta L_c^{\alpha}R_B^{\beta-1}$$

$$\Rightarrow (\frac{\lambda}{\alpha+\beta})Pc = L_c^{\alpha}R_B^{\beta} \qquad (7)$$

$$\therefore U(Lc, R_B) = (\frac{\lambda}{\alpha+\beta})Pc$$

For Karush-Kuhn-Tucker conditions to hold, lambda must be greater than zero.

In the legal negotiation, zero lambda means zero utility which is not acceptable for a rational player (Felipe, Adams, 2005). Subsequently, lambda must always be greater than zero. This means that when Lc increases we need to adjust R_B to maintain at least a positive utility. The easiest way to do that is to increase Pc, the expectation benefit. Thereby increasing either W_B or S_B.

When Lc, the transaction costs gets lower, lambda increases, thereby increasing the utility of the players, which is in conformity with Coase's Theorem (Cambridge Press, 2008).

However, when Lc is very low, say close to zero, lambda is nearly totally dependent on R_B which also increases due to the absence of or very low transaction costs. So lambda decreases resulting in lower utility.

Therefore, we can say that transaction costs cannot be too low or too high.

Optimal transaction cost algorithm

In a legal dispute, if we hold the attorney fees and other costs C_b in the reasonable bargain equation constant, then we can say that a function Φ of the transaction costs is dependent on Lc.

Let Li = $(L_1,\ldots\ldots,L_n)^T$, then $R_B + \Phi(Lci) \leq Pc$

$$\Rightarrow \phi(Lci) = \sum_{i=1}^{n}\phi i(Li) \qquad (8)$$

As shown in the previous section,

$$\phi(Lci) \neq 0; \phi(Lci) > 0; \phi(Lci) < R_B$$
$$\Rightarrow 0 < \phi(Lci) < R_B$$

A more realistic model of the transaction cost in the justice system based on transaction cost theory (Bylund, 2021) and (Reindfleisch, 2019) is that it should be proportional to the intensity of the case.

Let α_i^+ and α_i^- be the cost rates associated with Li=$(L_1,\ldots\ldots L_n)^T$, we can say that the equation below holds.

$$\phi i(Li) = \begin{cases} \alpha_i^+ Li, Li > 0 \\ -\alpha_i^- Li, Li < 0 \\ 0, Li = 0 \end{cases}$$

If we consider the fixed costs C_b we have;

$$\phi i(Li) = \begin{cases} C_b + \alpha_i^+ Li, Li > 0 \\ C_b - \alpha_i^- Li, Li < 0 \\ 0, Li = 0 \end{cases}$$

$$\Rightarrow \phi i(Li) = \begin{cases} 0, Li = 0 \\ C_b + \alpha_i \mid Li\mid, Li \neq 0 \end{cases}$$

Lambda from the Lagrangian equation describes the shadow values of the feasibility constant Pc.

$$\Rightarrow U(Lci, R_B) = (\frac{\lambda i}{\alpha+\beta})\left[\left(\sum_{i=1}^{n}\phi i(Li)\right) + R_B\right] \qquad (9)$$

The determinant of the bordered Hessian can be deduced as;

$$\det(H) = \begin{vmatrix} 0 & -\lambda p_1^* & -\lambda p_2^* \\ -\lambda p_1^* & \frac{\partial^2 L}{\partial L_C^2} & \frac{\partial^2 L}{\partial L_C \partial R_B} \\ -\lambda p_2^* & \frac{\partial^2 L}{\partial L_C \partial R_B} & \frac{\partial^2 L}{\partial R_B^2} \end{vmatrix} \qquad (10)$$

In terms of lambda and the feasibility constraint we have,

$$\det(H) = \begin{vmatrix} 0 & -\lambda p_1^* & -\lambda p_2^* \\ -\lambda p_1^* & -\lambda(\frac{\alpha}{\alpha+\beta})\frac{Pc}{L_C^2} & 0 \\ -\lambda p_2^* & 0 & -\lambda(\frac{\beta}{\alpha+\beta})\frac{Pc}{R_B^2} \end{vmatrix} \qquad (11)$$

resulting in,

$$\|H\| = 0 - (-\lambda p_1^*)\left[\left(-\lambda p_1^*\right)\left(-\lambda(\frac{\beta}{\alpha+\beta})\frac{Pc}{R_B^2}\right)\right] + (-\lambda p_2^*)\left[-\left(-\lambda p_2^*\right)\left(-\lambda(\frac{\alpha}{\alpha+\beta})\frac{Pc}{L_C^2}\right)\right]$$

In terms of the change in Lc and R_B from the utility function we have,

$$\det(H) = \begin{vmatrix} 0 & -\lambda p_1^* & -\lambda p_2^* \\ -\lambda p_1^* & \alpha(\alpha-1)L_C^{\alpha-2}R_B & 0 \\ -\lambda p_2^* & 0 & \beta(\beta-1)L_C^{\alpha}R_B^{\beta-2} \end{vmatrix} \qquad (12)$$

resulting in;

$$||H|| = 0 - \left(-\lambda p_1^*\right)\left[\left(-\lambda p_1^*\right)\left(\beta(\beta-1)L_C^\alpha R_B^{\beta-2}\right)\right] + \left(-\lambda p_2^*\right)\left[-\left(-\lambda p_2^*\right)\left(\alpha(\alpha-1)L_C^{\alpha-2}R_B\right)\right]$$

From the two equations resulting from det(H), we can see consistent signs ++ or --, so in order to obtain a local maxima we need a set of values of alpha that will make sure that the signs are consistent.

If $\alpha_p = (\alpha_{1,\ldots\ldots\ldots}\ \alpha_n)$ and $\beta_q = (\beta_1,\ldots\ldots\ldots,\beta_n)$ s.t. $\alpha \epsilon \alpha_p$ and $\beta \epsilon \beta_q$, then;

$$\exists \alpha^* \ s.t. (\frac{\lambda i}{\alpha+\beta}) > 0$$

Therefore this particular alpha which is a power of Lc in the utility function U(Lc, R_B) leads to the measure of the optimal transaction cost.

Thus $U^*(L^*c, R_B) = (\frac{\lambda^*}{\alpha^*+\beta})\left[L^*c + R_B\right]$ (13)

Conclusions

In this paper, we first discussed two big problems that arise when we label specific thresholds of transaction costs in the legal process as previously undertaken by scholars of law and economics. The first problem is that high transaction costs may induce cooperation while low transaction costs may induce non-cooperation which is in contrast with Coase's Theorem which states that lower transaction costs induce private agreements. The second problem is that since the reasonable bargain is built around the threat value, it does create a sense of fairness (i.e restitution plus projected payoff) to the parties involved. However, since our initial claim is that the legal process is a market, we need to put a price on the claim rather than just a reasonable bargain. This means the reasonable bargain is just the minimum mutually acceptable cooperative position.

We have shown how transaction costs influence the set of strategies played by players in a legal game, described the essence of a social utility function and how it could be maximized through an optimal transaction cost algorithm. We have also used Cobb-Douglas optimization to show how an optimal transaction cost algorithm can be deduced using the Lagrangian approach.

In most court cases, fees are intended to shift the costs of the justice system from taxpayers to litigants who are seen as the "users" of the courts. These costs may be used for covering parts of the justice process such as court-appointed attorney fees, court clerk fees, forensic fees, equipment fees, utilities etc. In this research we recommend that the courts should balance their costs between administration costs and taxpayers support for optimal distribution of social welfare.

Further research is needed to determine a more specific transaction cost algorithm based on empirical evidence or an econometric approach to complement the theoretical foundations derived in this paper. Finally, there is the need for further research into how psychological factors affect the probability of winning trial and the link between transaction costs and behavioral patterns in the legal process.

References:

1. Bonsu, K.O., Chen, S. (2020). Mechanism of Instrumental Game Theory in the Legal Process via Stochastic Options Pricing Induction. *Asian Research Journal of Mathematics, 16*(8), 152-173. https://doi.org/10.9734/arjom/2020/v16i830215
2. Bylund, P.L. (2021). The Firm versus the Market: Dehomogenizing the Transaction Cost Theories of Coase and Williamson. *Strategic Management Review, 2*(1), 79-118. http://dx.doi.org/10.1561/111.00000018
3. Felipe, J., Adams, F.G. (2005). "A Theory of Production" The Estimation of the Cobb-Douglas Function: A Retrospective View. *Eastern Economic Journal, 31*(3), 427-445.
4. Flores Borda, G. (2011). Game Theory and the Law: The "Legal-Rules-Acceptability Theorem" (A Rational for Non-Compliance with Legal Rules). *Derecho & Sociedad, 36,* 301-314. https://doi.org/10.1080/09672567.2021.1936713
5. Kim, J. (2013). Revisiting the Learned Hand Formula and Economic Analysis of Negligence. *Journal of Institutional and Theoretical Economics (JITE), 169*(3), 407-432. https://doi.org/10.1628/093245613X668601
6. Matthew, T.P. (2021). Progress Through Regression: The Life Story of the Empirical Cobb-Douglas Production Function. *The European Journal of the History of Economic Thought, 28*(4), 691-693.
7. McKelvey, R., Page, T. (1999). Taking the Coase Theorem Seriously. *Economics and Philosophy, 15*(2), 235-247. https://doi.org/10.1017/S0266267100003990
8. Rindfleisch, A. (2019). Transaction cost theory: past, present and future. *AMS Review, 10*, 85–97. https://doi.org/10.1007/s13162-019-00151-x
9. Sandler, I.L., Ivanov, D.V., Lifshits, M.Yu., Diligenskaya, A.N. (2021). Identification of generalized Cobb-Douglass production functions with multiplicative errors in variables. *IOP Conf. Series: Materials Science and Engineering, 1064.* https://doi.org/10.1088/1757-899X/1064/1/012015